

Illustration 1

The average percentage of failure in certain examination is 30. What is the probability that out of a group of 5 candidates atleast 4 passed in the examination?

Solution:

All the trials are independent. The number of pass in the examination may be minimum 4 (or) 5 all of them may pass.

$$p = 1 - q$$

$$q = \frac{30}{100} = 0.3$$

$$\therefore p = 1 - 0.3 = 0.7$$

\therefore Probability of passing 4 (or) more candidates is

$$P(X = x) = n_{cx} p^x q^{n-x}; \quad x = 0, 1, \dots, n$$

$$P(X \geq 4) = P(X = 4) + P(X = 5)$$

$$= P(4) + P(5)$$

$$= {}_5C_4 (0.7)^4 (0.3)^{5-4} + {}_5C_5 (0.7)^5 (0.3)^{5-5}$$

$$= \frac{5 \times 4 \times 3 \times 2}{1 \times 2 \times 3 \times 4} (0.7)^4 (0.3)^1 + 1 \cdot (0.7)^5 (0.3)^0$$

$$= 5(0.7)^4 (0.3) + 1 \cdot (0.7)^5 \cdot 1$$

$$= 0.36015 + 0.16807$$

$$= 0.52822$$

Illustration 2

The mean of a binomial distribution is 45 and the standard deviation is 6. Find the value of the parameters n & p .

Solution:

Given

$$\text{Mean} = np = 45$$

$$S.D = \sqrt{npq} = 6$$

$$\Rightarrow npq = 36$$

$$45q = 36$$

$$q = \frac{36}{45}$$

$$q = 0.8$$

$$\therefore p = 1 - q = 1 - 0.8 = 0.2$$

$$p = 0.2$$

$$\therefore n = \frac{45}{p} \quad [\because np = 45]$$

$$= \frac{45}{0.2}$$

$$n = 225$$

Illustration 3

Obtain a Binomial distribution for which mean is 15 and the variance is 6.

Solution

The mean of Binomial

$$\text{distribution} = np = 15$$

$$\text{Variance} = npq = 6$$

$$\therefore q = \frac{npq}{np} = \frac{6}{15}$$

$$q = 0.4$$

$$\therefore p = 1 - q = 1 - 0.4$$

$$p = 0.6$$

$$np = 15 \Rightarrow n = \frac{15}{p} = \frac{15}{0.6} = 25$$

Therefore, the required Binomial distribution is

$$(p - q)^n = (0.6 + 0.4)^{25}$$

(or)

$$P(X = x) = {}^{25}C_x (0.6)^x (0.4)^{25-x}$$

Exercise

1. The incidence of occupational disease in an industry is such that the workmen have a 25% chance of suffering from it. What is the probability that out of six workmen 4 or more will contract the disease?
2. African people infected with trypanosoma parasite are prone to get sleeping sickness with a chance of 75%. What is the probability that 48 out of 60 workmen will contract the disease?
3. The incidence of Wilson's disease in a particular area is such that the children have a 60% chance of getting diseased. What is the probability that out of 50, 30 or more will be affected?
4. In a lab, site directed mutagenesis was carried out. Around 25% change of mutation was predicted by the scientists. What is the probability that out of 20 sites 12 or more will be mutated?
5. The incidence of hysteria in mentally depressed patients due to stress is such that the patients have 73% chance of being hysteric. What is the probability that out of 83, 27 or more will be hysteric?
6. The incidence of diabetes mellitus with ketoacidosis shows lot of variations in Glucose, Aceto Acetate, bicarbonate, urea, potassium and creatinine levels at around 90% in children who are affected by it. What is the probability that out of 125 children 80 or more will show the varied levels?

Illustration 1

1. The screws produced by a certain machine was checked by examining 12 samples. The following table shows the distribution of 128 samples according to the number of defective items they contained.

No of defectives (in a sample of 12):	0	1	2	3	4	5	6	7	Total
No. of Samples:	7	6	19	35	30	23	7	1	128

Fit a Binomial distribution and find the expected frequencies if the chance of machine being defective is $\frac{1}{2}$. Find the mean and variance of the fitted distribution.

Solution

$$p = \frac{1}{2} ; q = \frac{1}{2}$$

$$P(0) = q^n = \left(\frac{1}{2}\right)^{12}$$

$$f(0) = N \cdot q^n = 128 \left(\frac{1}{2}\right)^{12}$$

Using the recurrence formula, the various probabilities $P(1)$, $P(2)$, ... can be calculated as follows.

x	$\frac{n-x}{x+1}$	$\frac{n-x}{x+1} \cdot \frac{p}{q}$	Expected Frequency $f(x) = N \cdot P(x)$
0	12	12	$f(0) = N \cdot P(0) = 0.03125$
1	5.5	5.5	$f(1) = 0.03125 \times 12 = 0.375$
2	3.33	3.33	$f(2) = 2.0625$
3	2.25	2.25	$f(3) = 6.868$
4	1.6	1.6	$f(4) = 15.45$
5	1.17	1.17	24.7
6	0.86	0.86	28.93

3. The juice prepared out of a specific herbal formulation for indigestion was checked by examining different samples. The following table shows the distribution of 195 samples according to the number of different ingredients they contained.

No of ingredients	1	2	3	4	5	6	7	8	Total
No of Samples	32	26	30	48	29	12	8	10	195

Fit a binomial distribution and find the expected frequencies if the chance of formulation being effective is $\frac{1}{6}$. Find the mean and variance of the fitted distribution.

4. A group of pregnant women of different age limit in a private hospital gave birth to their off springs. The table shows the distribution of 25 off springs according to their age distribution. Fit a binomial distribution.

Age:	20	22	24	26	28	30	32
No of off springs:	3	5	4	3	5	2	3

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